

1. (25%) Please answer the following questions.

(a) (10%) Consider the system, $\frac{d^2y}{dt^2} + 100y = 100r$, where r is the reference input and y is the output. Please find the transfer functions $G(s) = Y(s)/R(s)$ and the unit-step response $y(t), t \geq 0$.

(b) (10%) Consider the system shown in Fig. 1. Please find the transfer functions $G(s) = Y(s)/R(s)$ and the unit-step response $y(t), t \geq 0$.

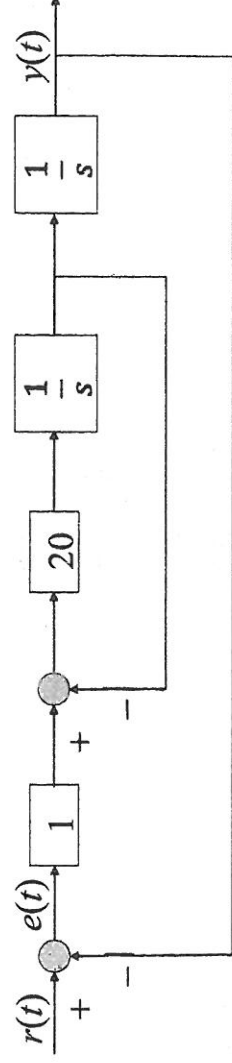


Fig. 1

(c) (5%) Find a first-order system whose unit-step response is a good approximate of that of the system of problem (b).

2. (25%) Consider the system shown in Fig. 2.

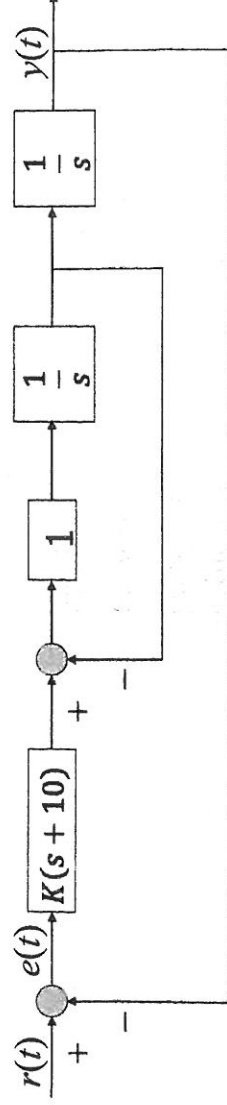


Fig. 2

- (a) (10%) Draw the root-locus of the system with respect to the gain K . Determine (i) number of branches, (ii) real-axis segments, (iii) starting and ending points, and (iv) asymptote(s) of the root locus. (Remember to state how you arrive at the answers.)
- (b) (5%) Determine the real-axis breakaway and break-in points of the root locus, if there are any.
- (c) (10%) Based on the root-locus determined by (a) and (b), describe how to select a gain such that the closed-loop poles satisfy the requirements of percent overshoot $\%OS \leq 5\%$ and 2%-settling time $T_{s,2\%} \leq 2$ sec and with minimum influence of the extra zero in the closed-loop transfer function. (Do not try to find a specific gain. Just describe how you will do in details.)

3. (50%) Consider a unity negative feedback system as shown in Fig. 3, where K is a proportional feedback gain. The Bode plot of the open loop system $G(s)$ is shown in Fig. 4. Please answer the following questions. Please provide reasons to your answers. No score will be given without reasons.
- (a) (10%) Please estimate the phase margin and gain margin of the system $G(s)$.
- (b) (10%) Please estimate the range of K such that the closed-loop system is bounded-input bounded-output (BIBO) stable.
- (c) (10%) Please estimate the bandwidth of the closed-loop system when $K=1$.
- (d) (10%) Please estimate the damping ratio of the closed-loop system when $K=1$.
- (e) (10%) Please estimate the steady-state error of the closed-loop system due to unit ramp input when $K=1$.

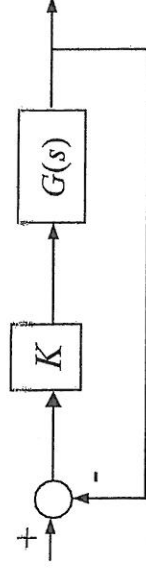


Fig. 3

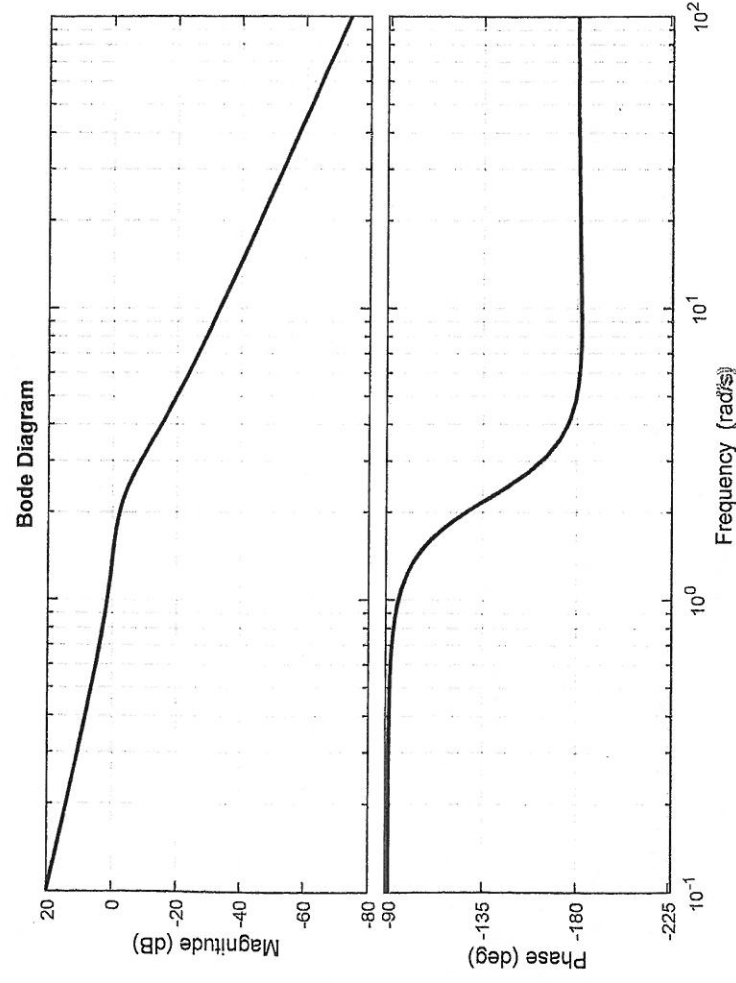


Fig. 4