

1. (15 points) Solve  $y' + y/x = \sin x$ , associated with the condition:  $y(\pi) = 1$ .
2. Solve the following second-order ordinary differential equation:  $y'' + 4y = x^2 \sin 2x$ .
  - (a) (10 points) Find the homogeneous solution.
  - (b) (10 points) Find the particular solution.
3. (15 points) Evaluate the Laplace transform of the following function:

$$\int_0^t \frac{\sin wt}{t} dt$$

4. (15 points) Compute  $\int_0^{\infty} \exp(-x^2) dx$ .

5. (a) (15 points) Solve  $u_t = \alpha u_{xx}$ , where  $\alpha$  is the diffusion coefficient, with the initial and boundary conditions.

$$\text{For } t = 0, u = 0.$$

$$\text{For } t > 0, u = 1 \text{ at } x = 0$$

$$u = 0 \text{ at } x = \infty$$

- (b) (5 points) If concentration at one location  $x = 0.1$  cm reaches a certain value at  $t = 3$  min, how long it takes to reach the same concentration at the other location  $x = 0.2$  cm.

6. (15 points) Solve the heat transfer of a hot metal rod described by

$$u_t = u_{xx}$$

$$\text{with IC: } u(x,0) = \sin(x) + 0.2 \sin(100x) + \frac{2}{\pi} x + 1, \text{ and BCs: } u(0,t) = 1, u(\pi, t) = 3.$$