國立中正大學九十學年度碩士班招生考試試題

系所別：機電光整合工程研究所　　科　目：工程數學

考生請注意：本試題分為三大部份，第一部份為微分方程，第二部份為線性代數，第三部分為複變，三個部份各含五十分的題目，請考生自行選擇二個部份作答，共一百分。考生必須在答案卷前註明所欲選擇的兩個部份。若有三個部份全選者，則第三部份不予以計分。

第一部份　微分方程　Differential Equations　(50%)

1. (a) Solve the following equation $2xy\frac{dy}{dx}+2y^2=3x-6$　(8%)

(b) Solve the following equation $\frac{d^2x}{dt^2}+4x=3e^{2t}+7\cos(t)$　(7%)

(c) Solve the following partial differential equation　(10%)

$$\frac{\partial^2 u}{\partial x^2}=4\frac{\partial^2 u}{\partial t^2}$$

subject to boundary condition

$$u(0,t)=u(2,t)=0$$

and initial condition

$$\begin{cases} u(x,0)=0 & 0<x<2 \\ u_t(x,0)=f(x) & 0<x<2 \end{cases}$$

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2. Figure 1 shows a simplified model of a car riding over a road surface.

(a) Derive the differential equation relating road profile x_i to car body displacement x_o. (6%)

(b) If $x_i(t) = \sin(t)$, find the general solution of x_o by the method of undetermined coefficients. (6%)

If we were to simulate the case of the car riding over a bump (see Figure 2), we can model the bump as half of a sine wave during the time interval [a,b] and with the amplitude A.

(c) Derive $x_i(t)$ using step functions. (6%)

(d) Assuming zero initial conditions, find the Laplace transform of $x_o(t)$. (7%)

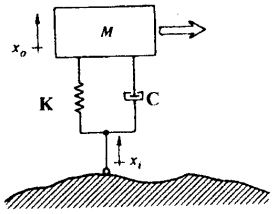

Fig. 1

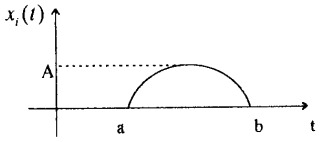

Fig. 2

第二部份　線性代數　Linear Algebra　(50%)

1. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}$ (10%)

2. Given $A = \begin{bmatrix} 6 & -10 \\ 3 & -5 \end{bmatrix}$, find A^{10} by the diagonalization method. (10%)

3. Prove that the eigenvalues of a Hermitian matrix are real. (10%)

4. Given a set of data points (1,1), (2,3), (3,4), (4,6), and (5,5), determine the equation of a best-fit line by the method of least squares. (10%)

5. Find out what type of conic section the following quadratic form represents and transform it to principal axes： (10%)

$$3x_1^2 + 4\sqrt{3}x_1x_2 + 7x_2^2 = 9$$

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第三部份 複 變 (Complex Variables) 50%

1. If z is a complex variable, evaluate each of the following using theorems of limits.

   (a) $\lim_{z\to -2i} \dfrac{(2z+3)(z-1)}{z^2-2z+4} = ?$ (5%)

   (b) $\lim_{z\to 2e^{i\pi/3}} \dfrac{z^3+8}{z^4+4z^2+16} = ?$ (7%)

2. $f(z) = \dfrac{z^8+z^4+2}{(z-1)^3(3z+2)^2}$

   (a) Locate and name all sigularities of $f(z)$. (5%)

   (b) Determine where $f(z)$ is analytic. (3 %)

3. Complex integration

   (a) If C is defined by $\pi^2 y = x^2$, evaluate $\int_C (z+2)e^{tz}dz$ from (0,0) to (π,1). (7%)

   (b) If $f(z)$ is analytic inside and on the boundary C of a simply connected region R, evaluate $\oint_C \dfrac{e^{2z}}{(z+1)^4}dz$. (8%)

4. Evaluate $\dfrac{1}{2\pi i}\int_{a-i\infty}^{a+i\infty} \dfrac{e^{zt}}{\sqrt{z+1}}dz$ where a and t are any positive constants. (15%)