國立中正大學九十一學年度碩士班招生考試試題

系所別：機電光整合工程研究所　　科　目：工程數學

1. Consider an L-C series circuit in which $E(t)=0$. Determine the current in the circuit for $t>0$, if its initial charge is q_0 and if initially there is no current flowing in the circuit. (10%)

2. Evaluate the inverse Laplace transform of $F(s)=\frac{1}{(s-1)(s+2)(s+4)}$ (10%)

3. (a) Prove that $\mathcal{L}\{f''(t)\}=s^2F(s)-sf(0)-f'(0)$ (5%)

   (b) Use the *Laplace* transform to solve the following differential equation (5%)

   $y''-5y'+6y=u(t-1); y(0)=0; y'(0)=1$

4. Find the eigenvalues and eigenvectors of (10%)

$$A=\begin{bmatrix} 1 & 2 & 2 \\ 2 & 3 & -2 \\ -5 & 3 & 8 \end{bmatrix}$$

5. The position of a moving particle is given by $\vec{r}(t)=2\cos t\hat{i}+2\sin t\hat{j}+3t\hat{k}$.

   Find the unit tangent vector $\vec{T}(t)$ and the curvature $k(t)$. (10%)

6. An engineering problem whose field equation is given by

$$\frac{\partial u(t,x)}{\partial t} = \frac{\partial^2 u(t,x)}{\partial x^2}. \qquad (1)$$

If boundary conditions at $x = 0$ and $x = 1$ for Eq. (1) are represented as

$u(t,0) = 0$ for $t > 0$

and

$u(t,1) = 0$ for $t > 0$,

and, the initial condition is given by

$u(0,x) = 1$ for $0 \le x \le 1$

(a) If the solution to this system is $f(t,x)$, find $f(t,x) = ?$ (20%)

If the solution to Eq. (1) with the modified boundary and initial conditions of

$$\begin{cases} u(t,0) = 1 & \text{for } t > 0 \\ u(t,1) = 0 & \text{for } t > 0 \\ u(0,x) = 0 & \text{for } 0 \le x \le 1 \end{cases}, \qquad (2)$$

is represented by $g(t,x)$ (You do not have to find $g(t,x)$).

(b) Propose a solution, in terms of $f(t,x)$ and $g(t,x)$, to Eq. (1) with the boundary and initial conditions of

$$\begin{cases} u(t,0) = 1 & \text{for } t > 0 \\ u(t,1) = 0 & \text{for } t > 0 \\ u(0,x) = 1 & \text{for } 0 \le x \le 1 \end{cases}.$$

Verify the solution you proposed is a reasonable one. (10%)

7. Find Laurent series about the indicated singularity for each of the two functions. Name the type of the singularity in each case and give the region of convergence of each series.

(a) $\dfrac{e^{2z}}{(z-1)^3}$; $z = 1$. (5%)

(b) $\dfrac{z - \sin z}{z^3}$; $z = 0$. (5%)

8. Evaluate $\dfrac{1}{2\pi i}\oint_C \dfrac{e^{zt}}{z^2(z^2 + 2z + 2)}dz$ around a circle C with equation $|z| = 3$. (10%)