

國立中正大學九十二學年度碩士班招生考試試題

系所別：機電光整合工程研究所 科目：工程數學

第 1 頁，共 3 頁

工程數學含「線性代數」、「微分方程」與「複變」三大部分，每部份各佔 50%，考生必須自行選擇其中兩大部分作答。

第一部份：線性代數 50%

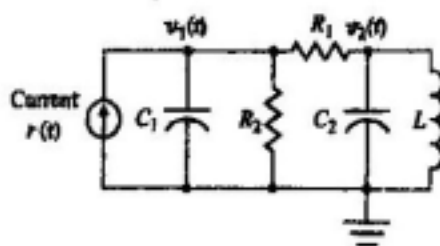
1. Find the eigenvalues and eigenvectors of $A = \begin{bmatrix} 9 & -10 & 2 \\ -6 & 5 & 2 \\ 6 & -2 & 1 \end{bmatrix}$

(10%)

2. $f(x) = 2x$ and $g(x) = 3 + cx$. If $f(x)$ and $g(x)$ are orthogonal to each other when $0 \leq x \leq 1$, find the normalized functions $\frac{f(x)}{\|f(x)\|}$ and $\frac{g(x)}{\|g(x)\|}$ (10%)

3. Find the eigenvalues and eigenfunctions of the following equation, $x^2 y'' + xy' + \lambda^2 y = 0$, $\lambda > 0$, and $y(1) = 0$; $y(e^\pi) = 0$. Show that this equation satisfies the Sturm-Liouville form. (15%)

4. The figure shows an electrical circuit. If current $i(t) = \delta(t)$ [impulse function], please find the Laplace transformation of $v_1(t)$ and $v_2(t)$ (15%)



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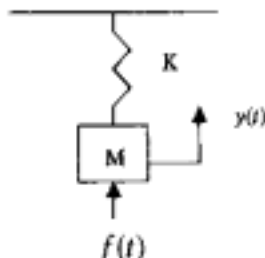
第 2 頁，共 3 頁

第二部份 微分方程 50%

1. Solve the following second-order non-homogeneous differential equation :

$$x^2 y'' - 3xy' + 4y = 12 \quad (10\%)$$

2. The figure shows an undamped spring-mass system. The excitation force is $f(t) = \cos(2t)$. The initial conditions are $y(0) = 0$ and $\frac{dy}{dt}(0) = 2$. If $M = 1$ and $K = 4$, please find the complete solution of $y(t)$ (15%)



3. Solve the partial differential equation (PDE)

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2}$$

subjected to the following initial and boundary conditions

$$f(x,0) = 0$$

$$f(0,t) = 1$$

$$f(1,t) = 0$$

either by the method of separation of variables **or** by the method of Laplace transformation. (25%)

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第 3 頁，共 3 頁

第三部份 複變 50%

- An analytic complex function $f(z)$ is represented by $f(z) = \phi + i\psi$, where $z = x + iy$. If $\phi = x^2 + 4x - y^2 + 2y$, find:
 - $\psi = ?$ (5%)
 - $f(z) = ?$ (Representing $f(z)$ in terms of the variable z) (5%)
- Prove that
 - $\sin(x + iy) = \sin x \cosh y + i \cos x \sinh y$ (8%)
 - $\frac{d\bar{z}}{dz}$ doesn't exist anywhere, where \bar{z} is the conjugate of z . (7%)
- For the following two functions, find the Laurent series about the indicated singularity and indicate the range of convergence for the series you obtained.
 - $z \cos \frac{1}{z}$; $z = 0$ (5%)
 - $\frac{1}{z(z+2)^2}$; $z = -2$ (5%)
- Evaluate
 - $\int_{-\infty}^{+\infty} \frac{dx}{x^4 + 1} = ?$ (6%)
 - $\int_0^{2\pi} \frac{d\theta}{5 + 3\sin\theta} = ?$ (9%)