Problem 4 (15%)

For 3×3 matrix A and B, $\det(A)=2$ and $\det(B)=-3$. Find (1) $\det(-3A)$, (2) $\det((A^{-2})^{T})$, (3) $\det(AA^{T})$, (4) $\det(BA)$, and (5) $\det(BAB^{-1})$.

Problem 5 (15%)

Consider matrix

$$A=\begin{bmatrix}3 & a & 2\\ 2 & b & 0\\ 2 & c & 4\end{bmatrix}$$

of which eigenvalues are found to be 0, 3, and 6.

(1) Find the values of a, b, and c in the matrix A. (5%)

(2) Find the characteristic equation of A. (2%)

(3) Find the eigenvectors of A. (5%)

(4) Show that the eigenvectors are linearly independent. (3%)

Problem 6 (15%)

(a) Show that $\int_{-\infty}^{\infty} f^2(t)dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}|F(\omega)|^2 d\omega$, where $F(\omega)$ is the Fourier transform of $f(t)$. (5%)

(b) Given a function $f(t)=e^{-at^2+i\omega_0 t}$ in time domain, where a is a positive constant and ω_0 is a constant angular frequency. Find the expression of the function in frequency domain. (10%)

Problem 7 (15%)

(1) Evaluate $\oint_C \frac{\sin(z)}{z^2(z^2+9)}dz$, where C encloses 0 and $3i$. (10%)

(2) Evaluate $\int_{1-i}^{3+2i} z^2 dz$ along the parabola $x=t$, $y=t^2$, where $1\le t\le 2$. (5%)

Problem 1 (20%)

Find the solution or the general solution for the following differential equations:

(a) $(2x^3+3y)dx+(3x+y-1)dy=0$ (5%)

(b) $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=0$, with the initial conditions of $y(0)=2$ and $\left.\frac{dy}{dx}\right|_{x=0}=-1$. (5%)

(c) $x^3\frac{d^3y}{dx^3}-x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}=0$ (5%)

(d) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=12xe^{2x}$ (5%)

Problem 2 (10%)

Find the general solution for the equation $2x^2\frac{d^2y}{dx^2}+3x\frac{dy}{dx}-(x+1)y=0$.

Problem 3 (10%)

Solve the following partial differential equation:

$$\frac{\partial Y}{\partial t}=k\frac{\partial^2 Y}{\partial x^2},\ (x>0,\ t>0)$$

subjected to the initial and boundary conditions as

$Y(x,0)=0$

$Y(0,t)=Y_0$

$Y(x,t)\to 0$, when $x\to\infty$

Where Y_0 is a constant.