

Problem 1 (20%)

Find the solution or the general solution for the following differential equations:

(a) $(4x + xy^2)dx + (y + x^2y)dy = 0$ for which $y(1) = 2$. (5%)

(b) $x \frac{dy}{dx} - 2y = x^3 \cos 4x$. (5%)

(c) $1 + y \frac{dy^2}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$. (5%)

(d) Solve the system:
$$\begin{cases} yzdx + xzdy + xydz = 0 \\ z^2(dx + dy) + (xz + yz - xy)dz = 0 \end{cases}$$
. (5%)

Problem 2 (10%)

Solve $\frac{d^2y}{dx^2} - 2x \frac{dy}{dx} + (x^2 + 2)y = e^{\frac{1}{2}(x^2+2x)}$.

Problem 3 (10%)

Solve the following partial differential equation

$$\frac{\partial^2 Y}{\partial t^2} = a^2 \frac{\partial^2 Y}{\partial x^2}, \quad (0 < x < l, t > 0)$$

subjected to the initial and boundary conditions of

$$Y(x, 0) = \mu x(1 - x)$$

$$\frac{\partial Y(x, 0)}{\partial t} = 0$$

$$Y(0, t) = 0$$

$$Y(l, t) = 0$$

Where μ is a constant.

Problem 4 (22%)

- (1) If $x(0) = 9$, $y(0) = 7$, and $z(0) = 0$, solve the following simultaneous differential equations. (15%)

$$\begin{aligned}\frac{dx}{dt} &= -4x + y + z \\ \frac{dy}{dt} &= x + 5y - z \\ \frac{dz}{dt} &= y - 3z\end{aligned}$$

- (2) For the following matrix,

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 2 & 0 \\ 2 & 0 & 4 \end{bmatrix}$$

find $|P(A)|$ if $P(x) = x^4 - x^3 - 3x^2 + 4x + 2$. (7%)

Problem 5 (15%)

Find the Fourier transform of $f(t) = \frac{e^{13t}}{t^2 - 4t + 13}$.

Problem 6 (23%)

(1) Evaluate $\int_{-\infty}^{\infty} \frac{x^2}{(x^2 + 1)(x^2 + 4)} dx$. (8%)

(2) Evaluate $\int_0^{\infty} \frac{\sin x}{x} dx$. (15%)