國立中正大學九十五學年度碩士班招生考試試題

系所別：光機電整合工程研究所　　科目：電磁學

第 3 節　　

1. Calculate the electric field $\vec{E}$, both inside and outside a uniform spherical charge distribution ρ, where R is the radius of the sphere.
   (a) (5%)Make use of the Gauss's law.
   (b) (10%)Calculate the electric potential by firstly solving the Poisson's equation:
   $$\nabla^2 V = -\frac{\rho(\vec{r})}{\epsilon_0} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$$
2. (a) (10%)Show that the electrostatic energy for the dielectric material can be written as:
   $$W = \frac{\epsilon_0}{2}\int \epsilon_r \vec{E}^2 d\tau$$
   where ϵ_r is the dielectric constant and $\vec{E}$ is the electric field.
   (b) (5%)With the result(a), calculate the energy stored in a dielectric-insulated plane parallel capacitor with capacitance C, where one of plates connected to ground and the other plate at a potential V.

3. Consider a **small** loop of wire of area S, situtated at the origin in a plane perpendicular to the z-axis and carrying a current I as shown in the figure. $\vec{r_1}$ is the unit vector along $\vec{r}$.

(a) (10%)Show that the vector potential $\vec{A}$ is given by

$$\vec{A} = \frac{\mu_0}{4\pi}\frac{\vec{m} \times \vec{r_1}}{r^2}$$

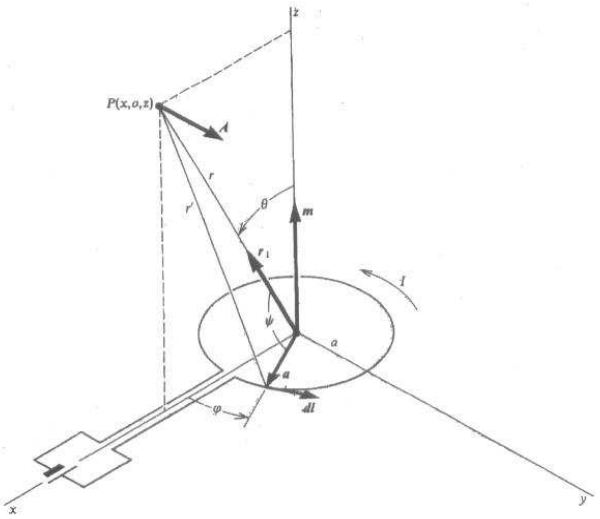
where $\vec{m} = I\vec{S}$ is called the magnetic dipole moment of the loop.

(b) (10%)From the result(a), show that the magnetic field can be expressed as :

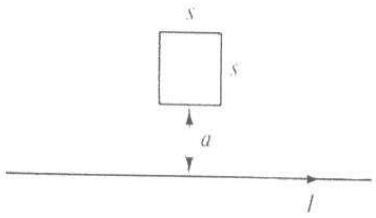
$$B_r = \frac{\mu_0}{4\pi}\frac{2m}{r^3}\cos\theta$$

$$B_\theta = \frac{\mu_0}{4\pi}\frac{m}{r^3}\sin\theta$$

$$B_\phi = 0$$



4. A square loop of wire (each side length = s) lies on a table near a very long straight wire which carries a current I (see the figure in below).
(a) Find the flux of magnetic field through the loop. (6%)
(b) If someone now pulls the loop directly away from the wire, at speed v, what emf (electromotive force) is generated? In what direction (clockwise or counterclockwise) does the current flow? (9%)
(c) What if the loop is pulled to the right at speed v, instead of away? (5%)



5. Suppose $V = 0$ (V : scalar potential), $\vec{A} = \hat{j}A_0 \sin(kx - \omega t)$ ($\vec{A}$: vector potential), where A_0, ω, and k are constants.
(a) Find electric field $\vec{E}$ and magnetic field $\vec{B}$. (6%)
(b) Check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω, and k? (14%)

6. Write down the electric and magnetic fields for a monochromatic plane wave of amplitude E_0, frequency ω, and phase angle zero that is
(a) traveling in the negative y-direction and polarized in the x-direction. (5%)
(b) traveling in the direction from the origin to the point (1,1,1), with polarization parallel to the xy plane. (5%)