

**Problem 1 (30%)**

Find the general solutions for the following differential equations,

$$(a) \frac{dy}{dx} = \frac{y \cdot \sin(x) - \sin(y)}{x \cdot \cos(y) + \cos(x) - y}$$

$$(b) y'' + 2y' + y = e^{-x} \cdot \ln(x)$$

$$(c) x^2 y'' - 4xy' + 6 = x^3 \cos(x)$$

**Problem 2 (20%)**

Solve the following differential equations by using Lapalce Transformation method,

$$(a) y'' + 4y = \sin(4t), y(0) = 1 \text{ and } y'(0) = 0$$

$$(b) y'' + y = u(t - \pi) - u(t - 2\pi), y(0) = 0 \text{ and } y'(0) = 1 \text{ where } u(t) \text{ is the unit step function.}$$

**Problem 3 (25%)**

Consider the following matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}.$$

$$(a) \text{ Find all eigenvalues and eigenvectors. (12\%)}$$

$$(b) \text{ Calculate } A^4 - 22A^2 - 24A - 55 = ? \text{ . (7\%)}$$

$$(c) \text{ Find } \det(AA^T) \text{ and } \det((A^{-2})^T). \text{ (6\%)}$$

**Problem 4 (25%)**

- (a) Solve the following first order differential equation by applying the Fourier transform

$$x' - 2x = H(t)e^{-2t}, -\infty < t < \infty$$

where  $H(t)$  is the unit step function or Heaviside function. (13%)

- (b) Evaluate  $\int_0^{\infty} \frac{\sin^2 x}{x^2} dx$ . (12%)