1. (25%) Consider the system shown in Fig. 1. The controller $K$ is a constant.

   (1a) (5%) Assume $K$ is equal to 100. Find the closed loop transfer function from $r$ to $y$.

   (1b) (5%) Determine the approximate rise time, and maximum overshoot of the output $y$ when $r$ is a unit step input.

   (1c) (5%) What is the system type? And what are the steady-state tracking errors due to a ***unit*** step and a ***ramp*** input ***r(t)=5t***, respectively?

   (1d) (10%) If you were asked to redesign the controller $K$ which should keep the rise time the same and reduce the overshoot, will you choose a lead or lag controller for the $K$? Explain why in details.

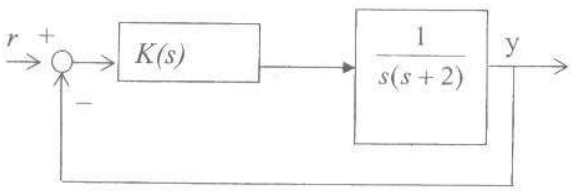



Fig. 1

2. (25%) Given a feedback system as shown in Fig. 2, where r is the reference input, y is the output, and e is the error. K is a constant gain.

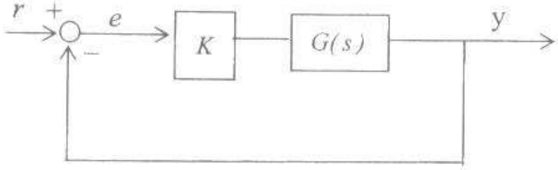



Fig. 2

The plant $G(s)$ is given as: $G(s)=\dfrac{2}{s^3+4s^2+3s}$

(2a) (5%) Determine the range of the K such that the closed loop is stable.

(2b) (10%) Plot the root locus of the plant. You should at least specify the following items in order to get full credits.

(1) The starting, ending and real-axis breakaway points

(2) The asymptotes

(3) The value of $K$ at which the root locus of $KG(s)$ crosses the imaginary axis.

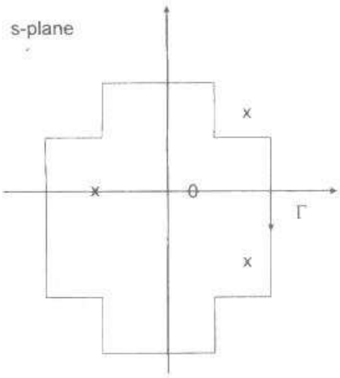
(4) The $j\omega$-axis crossing points

(2c) (5%) Determine how to use the root locus technique to determine the value of K such that the damping ratio of the closed loop poles (the complex conjugate pair) is 0.707.

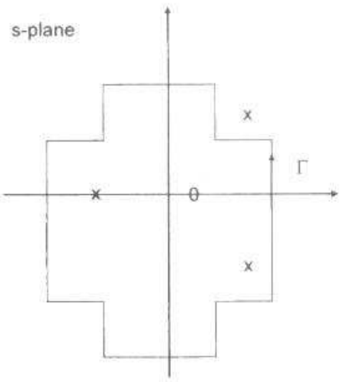
(2d) (5%) Explain why any K smaller than 0 will cause the closed loop system become unstable using Nyquist Criterion.

3. (10%) In each of the following two sub-problems, you are given a closed contour Γ and the locations of the poles and zeros of a transfer function G(s), as marked by 'x' and 'o'. Determine the number of CW (clockwise) encirclements of (0, 0) by G(Γ). Remember to describe how you obtain the answers. Be careful about the path direction of the contour Γ.

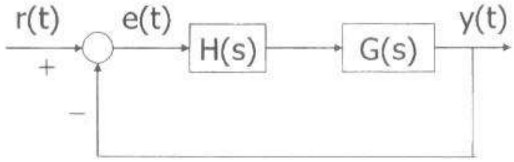
(3a) (5%)




(3b) (5%)




4. (40%) Consider the following feedback system where G(s) represents the plant dynamics and H(s) denotes a controller to be designed. Let $G(s) = \dfrac{1}{s^2(s+1)}$.




(4a) (10%) Sketch the Nyquist plot of G(.).

(4b) (10%) Use the Nyquist plot obtained in Problem (4a) to determine if it is possible to stabilize the closed-loop system when H(s) is a positive proportional (P) feedback controller, i.e., $H(s) = K_p$ and $K_p > 0$. Remember to explain how you arrive at the conclusion.

(4c) (10%) Consider a particular proportional-derivative (PD) feedback controller, $H(s) = K_p + s$ and $K_p > 0$. Apply the Nyquist analysis to determine the range of the $K_p$ gain over which the feedback system is stable.

(4d) (10%) Based on your result of Problem (4c), discuss how to use the frequency response approach to further refine and/or modify the PD controller such that the step response of the closed-loop system satisfies a desired percent overshoot requirement.