1. (a) (8%) Calculate the probability density current $\vec{S}(\vec{r},t)$ for a beam of electrons described by the sample wave function:

$\psi(\vec{r},t) = Ae^{i(\vec{k}\bullet\vec{r}-\omega t)}$

$\vec{S}(\vec{r},t)$ is defined as following: $\vec{S}(\vec{r},t) \equiv \frac{\hbar}{2mi}[\psi^*\nabla\psi - \psi\nabla\psi^*]$

(b) (7%) How does your result compare with the classical result for the current density of a beam of electrons having momentum $\vec{p} = \hbar\vec{k}$.

2. A particle with mass of m moves inside a one dimensional infinite square potential well with potential distribution as following:

$$V(x) = \begin{cases} 0 & \text{if } 0 \le x \le a \\ \infty & \text{otherwise} \end{cases}$$

(a) (10%) Calculate its eigen-energy and eigen-function for the nth stationary state.

(b) (10%) Calculate the expectation values of $\langle x \rangle$ and $\langle p \rangle$ for the nth stationary state.

(c) (15%) Assume the particle has the initial (t=0) wave function

$$\psi(x,0) = \begin{cases} Ax(a-x) & 0 \le x \le a \\ 0 & \text{otherwise} \end{cases},$$

where A is constant.

Find (i) the A constant, (ii) the time dependent wave function $\psi(x,t)$.

3. (10%) The wave function of a 1s electron is $\psi = \frac{e^{-r/a_0}}{\sqrt{\pi}a_0^{3/2}}$. Verify that the average value of 1/r for a 1s electron in the hydrogen atom is $1/a_0$.

4. (20%) This problem concerns with the photoelectric effect.

(a) Using the conservation laws of energy and momentum, show that a free electron (i) cannot completely absorb the energy of a photon; (ii) cannot emit energy

(b) When photons of wavelength 404.6 nm shine on a certain metal surface, the most energetic photoelectrons are stopped by a retarding potential of 1.6 V. For photons of wavelength 576.9 nm, the stopping potential is 0.45 V. Assuming h and e is unknown, what are the work function of the metal surface and the value of h/e? (h is the Planck's constant)

5. (20%) For particle statistics, there are three distribution functions: Maxwell-Boltzmann, Bose-Einstein, and Fermi-Dirac distribution. Please fill in the blanks with each property:

	Maxwell-Boltzmann	Bose-Einstein	Fermi-Dirac
Applies to systems of	Identical, distinguishable particles		
Category of particles	Classical		
Properties of particles	Any spin, particles far enough apart so wave function do not overlap		
Examples	Molecules of a gas		
Distribution function (number of particles in each state of energy ε at the temperature T)	$f_{MB}(\varepsilon) = Ae^{-\varepsilon/\kappa T}$		