1. Suppose the electric potential is given by

$$V(\vec{r}) = A\frac{e^{-\beta r}}{r}$$

for all r (A and β are constants).

(a) (6%) Find the electric field $\vec{E}(\vec{r})$.

(b) (8%) Find the charge density $\rho(\vec{r})$.

(c) (6%) Find the total charge Q.

2. A very long cylinder, of radius R, carries a uniform polarization $\vec{P}$ perpendicular its axis.

(a) (8%) Find the electric filed in side the cylinder.

(b) (12%) If the cylinder is placed in an otherwise uniform electric field $\vec{E}_0$, ($\vec{E}_0 \parallel \vec{P}$) find the resulting electric field within the cylinder (assume the electric susceptibility is χ_e.

3. A steady current $\vec{I}$ flows down a long cylindrical wire of radius R. Find the magnetic filed, both in inside and outside the wire, if

(a) (4%) The current is uniformly distributed over the outside surface of the wire.

(b) (6%) The current is distributed in such a way that J (current density) is proportional to r, the distance from the axis.

4. (10%) From Maxwell equations
$$\begin{cases} \nabla \cdot \vec{E} = \dfrac{\rho_V}{\varepsilon_0} \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} \\ \nabla \cdot \vec{B} = 0 \\ \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \dfrac{\partial \vec{E}}{\partial t} \end{cases}$$

Show that there exist a scalar potential V and vector potential $\vec{A}$ with
$$\begin{cases} \vec{E} = -\nabla V - \dfrac{\partial \vec{A}}{\partial t} \\ \vec{B} = \nabla \times \vec{A} \end{cases}$$

Such that both potentials satisfied the nonhomogeneous wave equations
$$\begin{cases} \nabla^2 \vec{A} - \varepsilon_0 \mu_0 \dfrac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0 \vec{J} \\ \nabla^2 V - \varepsilon_0 \mu_0 \dfrac{\partial^2 V}{\partial t^2} = -\dfrac{\rho_V}{\varepsilon_0} \end{cases}$$

5. (20%) For the uniform plane wave defined by
$$\begin{cases} \vec{E} = [\hat{a}_x + E_y \hat{a}_y + (2 + j6)\hat{a}_z] e^{-j2.3(-0.3x+0.4y)+jwt} \\ \vec{H} = [H_x \hat{a}_x + H_y \hat{a}_y + H_z \hat{a}_z] e^{-j2.3(-0.3x+0.4y)+jwt} \end{cases}$$

where H_x, H_y and H_z, E_y are all independent of x, y, and z, please determine follow items:

(a) The components E_y, H_x, H_y and H_z, assuming that $\mu = \mu_0$ and $\varepsilon = 4\varepsilon_0$, $\mu_0 = 4\pi \times 10^{-7}$ h/m and $\varepsilon_0 = 8.85 \times 10^{-12}$ f/m.

(b) The frequency and corresponding wavelength.

(c) The equation of the surface of constant phase.

(d) The state of polarization of the wave.

6. (20%) A linearly polarized plane wave from the vacuum strikes on a xy-plane interface of lossless dielectric ε_1 (refractive index n_1) at an incident angle θ_i.

(a) If the incident electric filed is $\vec{E}(x,y,z,t) = \hat{a}_x E_0 \exp(-j\vec{k}_0 \cdot \vec{R}) \cdot \exp(j\omega t)$, and the complex reflection coefficient, transmission coefficient, refracted angle and vacuum wave number are given as Γ, τ, θ_t, and k_0, respectively. Using these parameters to write down reflected and transmitted electric fields $\vec{E}_r, \vec{E}_t$.

(b) Find the incident magnetic field $\vec{H}_i$, reflected magnetic filed $\vec{H}_r$, and transmitted magnetic field $\vec{H}_t$ with additional parameters of coordinate unit vectors $\hat{a}_x, \hat{a}_y, \hat{a}_z$ and the intrinsic impedances η_0 and η_1. ($\eta_1 = \sqrt{\dfrac{\mu}{\varepsilon_1}}, n_1 = \sqrt{\dfrac{\varepsilon_1}{\varepsilon_0}}$)