1. (50%) Consider a control system with plant $G(s)=\frac{1}{s^2-1}$ and controller $C(s)$, as shown below. Three possible controllers are considered:

(i) Proportional controller (P-control): $C(s)=k, \quad k\geq 0$;

(ii) Proportional-derivative controller (PD-control): $C(s)=k(s+2), \quad k\geq 0$;

(iii) Proportional-integral controller (PI-control): $C(s)=\frac{k(s+0.5)}{s}, \quad k\geq 0$

It is desired that the closed-loop system satisfies the following requirements:

(R1) the stability is bounded-input-bounded-output (BIBO) stable

(R2) percent overshoot is less than 10%;

(R3) settling time is less than 2 sec.

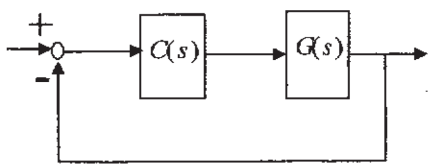
Please use the method of root locus to answer the following questions.

1.a (15%) Please determine the conditions on the locations of the closed-loop poles that can meet all of the requirements of (R1), (R2), and (R3). That is, what kind of closed-loop poles should be designed in order to satisfy the requirements?

1.b (5%) Please show that the plant is unstable.

1.c (15%) For the above three controllers, please determine those that meet the requirement (R1). That is, find the controllers that can stabilize the system.

1.d (15%) For those controllers that can stabilize the system (i.e., the answer given by problem 1.c), please determine if they can also meet the requirements (R2) and (R3).



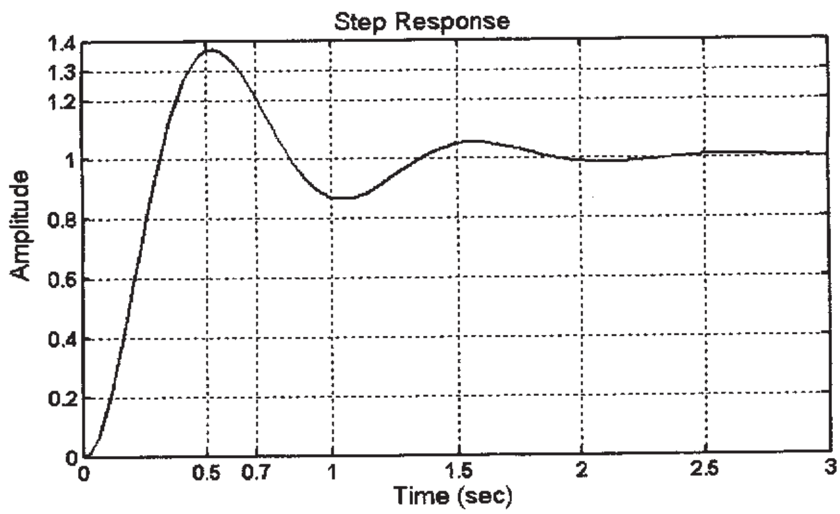
2. (15%) Consider the following 2^{nd} order system with two real poles, -1 and –p, and $p > 1$.

$$G(s) = \frac{p}{(s+1)(s+p)}$$

2.a (10%) Use the Laplace transform approach to derive its unit step response.

2.b (5%) Discuss when the unit step response of G can be approximated by the unit step response of a first order system, i.e, when the 2^{nd} order system can be treated as a dominantly 1^{st} order system. Furthermore, what is that approximate 1^{st} order system?

3. (35%) Consider the standard 2^{nd} order system $G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}, 0 < \zeta < 1$. The following figure is the unit step response of a G(s) with particular damping ratio ζ and natural frequency ω_n.



3.a (5%) Use the Laplace transform approach to verify the following expression of the unit step response of G. Note if you are running out of time, you may want to skip this question first.

$$y(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \cos\left(\omega_n\sqrt{1-\zeta^2}\, t - \phi\right) \quad \text{where} \quad \phi = \tan^{-1}\left(\frac{\zeta}{\sqrt{1-\zeta^2}}\right)$$

3.b (15%) No matter if you are successful in deriving the unit step response in problem 3.a, use the expression of the unit step response to show that the peaks of the unit step response occur at $t_k = \dfrac{k\pi}{\omega_n\sqrt{1-\zeta^2}}$, $k = 1, 2, 3, \ldots$ and the corresponding peak values are $y(t_k) = 1 - \exp\left(-\dfrac{k\zeta\pi}{\sqrt{1-\zeta^2}}\right)\cos(k\pi)$, $k = 1, 2, 3, \ldots$

3.c (10%) Consider the unit step response plot shown at the beginning of this Problem. Use its percent overshoot %OS and peak time t_p to estimate ζ and ω_n of the corresponding G(s) of the unit step response plot. You may need the following table of logarithm function ln(x).

x	1	0.35	0.36	0.37	0.38	0.39
ln(x)	0	-1.0498	-1.0217	-0.9943	-0.9676	-0.9416

3.d (5%) Discuss how to use t_k and $y(t_k)$, $k = 1, 2, 3, \ldots$ obtained in problem 3.b to improve the reliability of the estimated ζ and ω_n.