1. We assume frame S' moves at velocity v along the x axis of frame S. The y and z coordinates of an event will be the same in both frames, that is, $y' = y$ and $z' = z$. The x and t coordinates are related to the Lorentz transformation:

$$x' = \gamma(x - vt) \tag{1}$$

and

$$t' = \gamma(t - vx/c^2) \tag{2}$$

with $\gamma = (1 - v^2/c^2)^{-1}$.

(a) Derive the inverse Lorentz transformation $x = \gamma(x' + vt')$ and $t = \gamma(t' + vx'/c^2)$ from eqs.(1) and (2). (5%)

(b) Explain that the Lorentz transformation leads to time dilation and length contraction. (10%)

2. In the three dimensional time independent Schrodinger equation

$$-\frac{\hbar^2}{2m}\nabla^2\psi(x,y,z) + V(x,y,z)\psi(x,y,z) = E\psi(x,y,z) \tag{3}$$

$V(x, y, z)$ and E respectively denote the potential and total energy of the system under consideration and $\psi(x, y, z)$ is the associated wave function.

(a) Consider a free particle is confined in a cubic box with three sides directing along x, y and z axes respectively and covering the range between 0 and L. Derive the three lowest eigenenergy levels of this system (composed of the particle and the box) and the associated eigenfunctions. (5%)

(b) Will reduction of the box volume ($V = L^3$) increase or decrease the eigenenergy levels? (5%)

(c) Place N identical and distinguishable particles in the same box as mentioned in (a) and (b). Discuss how external energy supplied to the system (composed of the particles and the box) vary the distribution of particles on the eigenstates. Explain either work or heat shifts the eigenenergy levels additionally. Note that both work and heat are external energy. (10%)

3. Please give the orders of magnitude of (i) the radius of a hydrogen atom, (ii) the typical carbon-carbon bond-lengths, (iii) the diameter of a DNA double-helix, (iv) the size of particles (or clusters) studied in the fields of nanotechnology and nanoscience, and (v) the diameter of a hair. (10%)

4. STM (scanning tunneling microscope) and AFM (atomic force microscope) are two early versions of scanning probes that launched nanotechnology. Please briefly introduce any two later versions of microscopes useful in nanotechnology and nanoscience. (5%)

5. There are four types of crystalline solids. Ionic type is one of crystalline solids. Its lattice is composed by negative and positive ions. The bonding method is electric attraction. The cohesive energy is hard that is the work needed to remove an atom from the crystal. The properties of this type crystalline solid are high melting points, may be soluble in polar liquids such as water, and electrical insulators. The example is the sodium chloride (NaCl). According to the above description, explain the remaining three crystalline solids as lattice composition, bonding method, properties, and example. (15%)

6. Explain the following items:

a. Debye theory (5%)
b. Equipartition of energy (5%)
c. Hund's rule (5%)
d. Impurity semiconductors (5%)

7. In the free electron model of a metal, what are the assumptions of the electron model? Derive Fermi energy and the electron distribution function f (E). (15%)