

1. (20%) The block diagram of a feedback control system is shown in Figure 1.

(a) Derive the following transfer functions:

$$\frac{Y(s)}{R(s)} \Big|_{N=0} \quad \frac{Y(s)}{N(s)} \Big|_{R=0}$$

(b) The controller with the transfer function $G_d(s)$ is for the reduction of the effect of the noise $N(s)$. Find $G_d(s)$ so that the output $Y(s)$ is totally independent of $N(s)$.

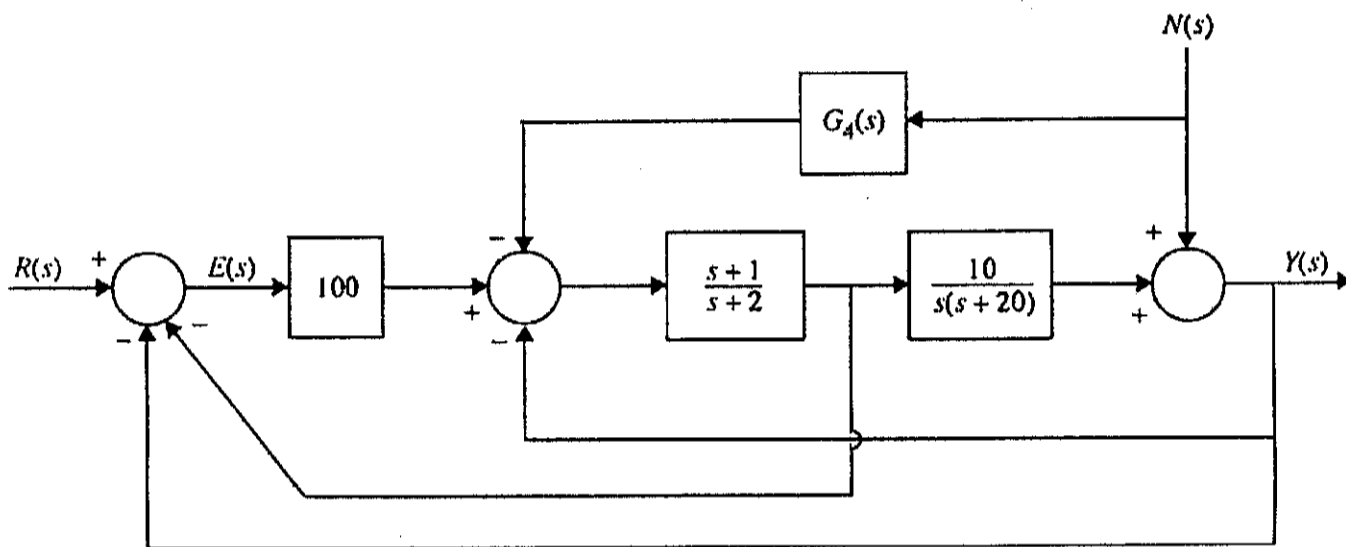


Fig. 1

2. (20%) A controlled process is represented by the following dynamic equations:

$$\frac{dx_1(t)}{dt} = -x_1(t) + 5x_2(t)$$

$$\frac{dx_2(t)}{dt} = -6x_1(t) + u(t)$$

$$y(t) = x_1(t)$$

The control is obtained through state feedback with

$$u(t) = -k_1 x_1(t) - k_2 x_2(t) + r(t)$$

where k_1 and k_2 are real constants, and $r(t)$ is the reference input.

- (a) Find the value of k_1 and k_2 such that $\xi=0.707$ and $\omega_n=10$ rad/sec.
- (b) Let the error signal be defined as $e(t) = r(t) - y(t)$. Find the steady-state error when $r(t) = u_s(t)$ and k_1 and k_2 are at the values found in part (a).
3. (10%) Using the Routh-Hurwitz criterion, determine how many roots are to the right of the line $s = -1$ in the s-plane for the closed-loop system that has the following characteristic equation

$$s^3 + 4s^2 + 4s + 4 = 0$$

4. (10%) Consider the following system with a P (proportional) feedback control.

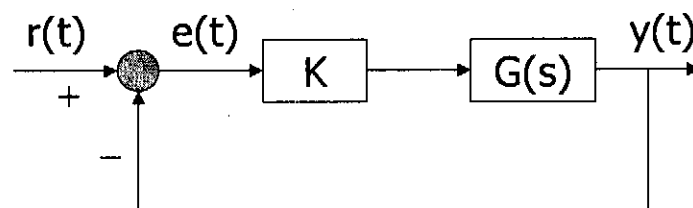


Fig. 2

where $G(s) = \frac{1}{(s+1)(s+2)(s+10)}$ and $K > 0$.

Sketch its root locus step by step by answering the following questions.

- (2%) Identify the real-axis segments of the root locus.
 - (3%) Determine the asymptotes.
 - (3%) Determine the breakaway point.
 - (2%) Determine the breakaway angles.
5. (40%) Consider the feedback system of Problem 4 again.
- (2%) Verify that the point $(-0.7 + j3.92)$ is on its root locus. Note that $\tan^{-1}(3.92/0.3) \cong 85.6^\circ$, $\tan^{-1}(3.92/1.3) \cong 71.6^\circ$, and $\tan^{-1}(3.92/9.3) \cong 22.8^\circ$.
 - (8%) Determine the K gain and closed-loop transfer function corresponding to the point $(-0.7 + j3.92)$ of the root-locus as verified in (a).
 - (5%) Determine the steady-state error in the unit-step response of the resultant closed-loop system of (b).
 - (10%) Discuss that the resultant closed-loop system of (b) is a dominantly 2nd order system.
 - (10%) Discuss the unit-step response of the dominantly 2nd order system of (d) in terms of the peak time, percent overshoot, and settling time within 2%.
 - (5%) Discuss how a PI- (proportional-integral) feedback control can remove the non-zero steady-state error in the unit-step response of the original P-feedback system.