1. Solve $x^3y' + 4y = x^3 + 2x$, $y(1) = e + 1$ (10%)

2. Solve $y' + \frac{1}{x}y = 3x^2y^3$ (10%)

3. If $\vec{A} = (3x^2 + 6y)\vec{i} - 14yz\vec{j} + 20xz^2\vec{k}$, evaluate $\int \vec{A} \cdot d\vec{r}$ from (0, 0, 0) to (1, 1, 1) along the following path c : $x = t$, $y = t^2$, $z = t^3$ (10%)

4. Solve by Laplace transform,
   $x' - 2y' = 1$, $x' + y - x = 0$, $x(0) = 0$, $y(0) = 1$ (10%)

5. In complex analysis, find $\cos(5i + 3)$ (10%)

6. For a scalar function f and two differentiable vector functions $\vec{E}$ and $\vec{H}$, please prove

(a) $\nabla \cdot (\vec{E} \times \vec{H}) = \vec{H} \cdot (\nabla \times \vec{E}) - \vec{E} \cdot (\nabla \times \vec{H})$ (10%)

(b) $\nabla \times (f\vec{E}) = f(\nabla \times \vec{E}) + (\nabla f) \times \vec{E}$ (15%)

7. Find the eigenvalues and eigenvectors of the so-called Pauli spin matrices and show that $S_xS_y = iS_z$, $S_yS_x = -iS_z$, $S_x^2 = S_y^2 = S_z^2 = I$,
   where

$$S_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ S_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},\ S_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \text{ (15\%)}$$

8. Find the inverse Laplace transform of the given function.

(a) $\frac{\omega\cos\theta + S\sin\theta}{S^2 + \omega^2}$ (5%)

(b) $\frac{3S^2 - 2S - S}{(S-3)(S^2+1)}$ (5%)