

1. Find a general solution of the differential equation, $y'' + 4y = \cos 3x$. (10%)
2. Calculate the inverse of the following matrix by the Gauss-Jordan elimination. (10%)

$$\begin{bmatrix} -1 & 1 & 1 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

3. Find a basis of eigenvectors for the following matrix. (10%)

$$\begin{bmatrix} 8 & -1 \\ 5 & 2 \end{bmatrix}$$

4. Prove the eigenvalues of a symmetric matrix are real. (10%)
5. Find the directional derivative of $f = x^2 + y^2$ at the point $P:(1, 1)$ in the direction of the vector $\mathbf{a} = 2\mathbf{i} - 6\mathbf{j}$ (10%)
6. Solve by Laplace transform, $L\left[\frac{1}{t} \sin at\right] = ?$ (10%)
7. $\begin{cases} x' + 2x - y' = 0, & x(0) = 0 \\ x' + y + x = t^2, & y(0) = 0 \end{cases}$, solve by Laplace transform (10%)
8. Find the Fourier cosine series of $f(x) = |x|$, $-l < x < l$ (10%)
9. Find (1) i^i , (5%)
(2) $(1+i)^3$ (5%)
10. $u = \cosh x \cdot \cos y$, find the analytic function $f(z) = u + iv$ (10%)