

1. Find a general solution of the following differential equations.
 - (a) $y'' - 2y' + y = e^x \sin x$ (5%)
 - (b) $y'' + y' + y = \cos x + 13 \cos 2x$ (5%)
 - (c) $y'' + 2y' + y = e^{-x} \cos x$ (5%)
 - (d) $xy'' - 4xy' + 6y = x^4 \sin x$ (5%)

2. Let $f = 4xyz$, $g = x^4 + y^4 + z$, $\vec{v} = x^2\vec{i} + (y-z)^2\vec{j} + xy\vec{k}$, $\vec{w} = (x+y)^2\vec{i} + z^2\vec{j} + 2yz\vec{k}$ (Assume the coordinate system to be right-handed whenever this is essential.) Find
 - (a) $\text{div}(\text{curl}(\vec{v} + \vec{w}))$ (5%)
 - (b) $\text{curl}(\text{grad } g) \cdot \vec{v}$ (5%)
 - (c) $\text{grad}(\text{div } \vec{w}) \cdot \vec{v}$ (5%)
 - (d) $\text{curl}(f(\vec{i} + \vec{j}))$ (5%)

3. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & -2 & 3 \\ -2 & -1 & 6 \\ 1 & 2 & 0 \end{bmatrix}$ (10%).

4. Solve the equation by Laplace Transform method, $L^{-1}\left[\tan^{-1}\frac{1}{s}\right]$. (10%)

5. Solve the partial differential equation, $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = x^2 + 2e^{2x} - 1$. (10%)

6. Transform the function $f(x) = \begin{cases} x & \text{if } 0 < x < a \\ 0 & \text{if } x > a \end{cases}$ in the form of Fourier sine integral. (10%)

7. Express the values of the following complex numbers in the form of $a + ib$, where a, b are real.
 - (1) $(2i)^{3i}$ (10%)
 - (2) $(1+i)^{1-i}$ (10%)